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A Method of Measuring Small Capacities by A Neon-Tube

By Tatsumi Terada

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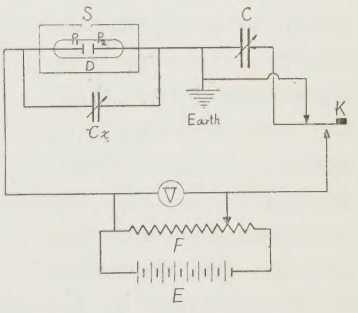
Abstract

The paper describes a simple method of measuring small capacities to the order of a few micro- microfarads by connecting a battery, a condenser, a Morse key and a neon-tube in series, with the addition of a voltmeter which is in parallel with the battery.

In Fig. 1, C and C_x are two condensers in series, C_x being in parallel with a neon-tube D of low pressure, and for the sake of convenience the tube D is enclosed in an ebonite box having a narrow slit S. K is a Morse key, E a battery of a few hundred volts, F a potentiometer and V a voltmeter.

Fig. 1

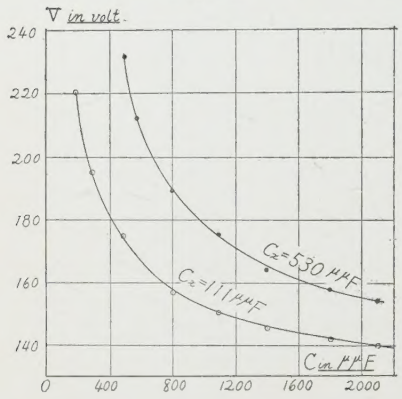


If the Morse key K is depressed the condenser C is charged by the battery E. When the applied voltage V is sufficient, a "flash" which is clearly seen through the slit S, appears in the tube D at the same time as the circuit is closed, but when the key K is released it does not appear and the terminals of the condenser C are simply shunted. The minimum voltage V that must be applied to cause a "flash" depends upon the capacity of the condensers C and C_x, that is, it increases with the capacity C_x when C is kept constant, while it decreases with the increase of the capacity C when C_x is constant.

Keeping C_x constant–530 and 111 micro-microfarads-respectively, the writer measured minimum voltages V causing a "flash" to appear in the neon-tube with various values of the capacity C. From these results the curves could be plotted as shown in Fig. 2, C and V being taken as the abscissa and ordinate respectively. It may be seen from the curves that the relation between the capacity C and minimum applied voltage V is represented by a hyperbola.

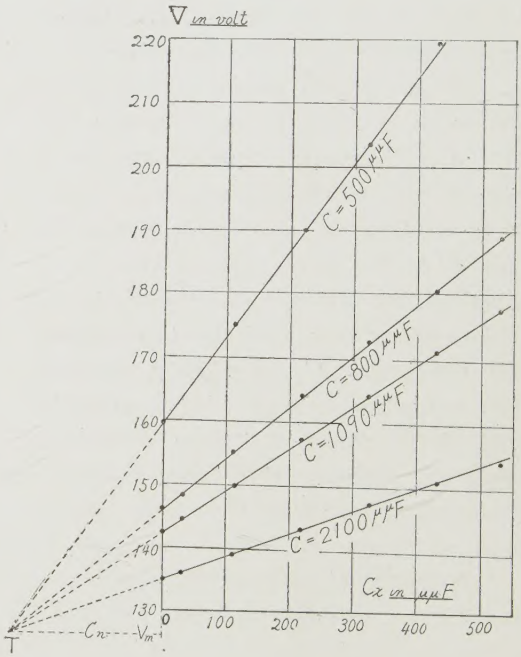
In Fig. 3 were plotted the minimum voltages V applied to cause a "flash" corresponding to various values of C_x which were obtained when the capacities of the condenser C were kept at constant values of 2100, 1090, 800 and 500 micro-microfarads respectively. From these results it is seen that the relation between V and C_x is represented by straight lines, all intercepting at a point T, the coordinates of the point being $V=126$ volts and $C_x=250$ micro-microfarads. Here it is to be added that the value of 126 volts is equal to the sparking potential V_m and 250 micro-micro-farads is the apparent capacity of the neon-tube as will appear later in the discussion.

Fig. 2



The "flash" of the neon-tube will occur when the potential difference between the electrodes P_1 and P_2 of the neon-tube in Fig. 1 reaches the sparking potential V_m. If we represent the capacity between the electrodes of the neon-tube by C_n and the total potential difference applied to the circuit by V, then the potential difference V_n between the electrodes of the neon-tube will be given by the formula

Fig. 3



$$V_n = V\frac{C}{C+C_x+C_n} \qquad \text{.....(1)}$$

Thus, the flash discharge appears in the tube only when this value of V_n becomes equal to the sparking potential V_m: that is

$$V_m = V\frac{C}{C+C_x+C_n},$$

or rewriting

$$V = V_m\left\{1+\frac{C_x+C_n}{C}\right\}. \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

From this equation, it is seen that when the capacities C_x and C_n are kept at constant values, the relation between V and C should be the hyperbolic curve as shown in Fig.2, and that when the capacities C and C_n are constant, the relation between V and C_x must be linear as indeed, they appear in Fig.3. Moreover, it is easily seen that the coordinates of the intercepting point T of these lines must be $-C_n$ and V_m, where C_n is the apparent capacity of the tube, whose value has been found from Fig.3 to be 250 micro-microfarads, V_m being the sparking voltage (126 volts). Consequently, the curves $V-C_x$ are obtained by joining the point T and a point on the V-axis, which corresponds to the minimum flash voltage for any value of C and $C_x=0$. Using the lines thus obtained, we can measure the various values of the capacity of the condenser. For example, the writer could easily measure a small capacity of the order of 3 micro-microfarads with the line of $C=1090$ micro-microfarads, and capacities of the order of 1.5 micro-microfarads or less by means of the line of $C=500$ micro-microfarads. The values thus measured are of an order comparable with those obtained by L. Hartshorn[1] with a bridge-arrangement.

A special point in the method described above is its being very simple as compared with the other one, though its accuracy mainly depends upon that of the voltmeter and also the electric leakage between the terminals of the condensers. When the leakage at the terminals of the condenser C is much more marked periodic "flashing"[2] will be repeated in the tube with voltages lower than those required for the single flash already described.

In conclusion the writer wishes to express his sincere thanks to Professor U. Yoshida for the interest he has taken in the research.

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1. L. Hartshorn: Phys. Soc. Proc. **36**, 399 (1924).
2. T. Terada: These Memoirs, **17**, 85 (1934); **17**, 381 (1934).

A Note on Equations for the Meson

By Hideo Yamamoto

(Received January 26, 1940)

In a previous paper[1] I demonstrated how I arrived at generalized equations for the Dirac electron by means of the usual tensor calculus which is familiar to us in the relativity theory of gravitation. Since there no spin matrix can be introduced in the tensor formulations as is done for the spinor ones, it seems difficult to describe the spinning electron by the tensor formulations. I have, however, further shown[2] that the spin momentum can be introduced in the tensor formulations as a certain operator which can be applied to vectors of the wave-field, and I have drawn the conclusion that spinor formulations are not necessarily needed to describe the spinning electron.

One of the advantages of the tensor formulations is that the formulations enable us to make geometrical considerations for the wave-fields of the electron without use of any auxiliary spaces: in the previous paper[1] I developed a scheme for a world geometory to describe the physical world which consists of space, time, and electron.

In this note I shall briefly describe a method of obtaining a tensor formulation of the so-called U-field of Yukawa's theory[3] by means of the tensor calculation which I used in the previous papers, and shall show a possible scheme of unifying the representations of the Dirac and Yukawa theories.

Let us consider the second order wave equation of Schrödinger's type with respect to a vector $\psi^{\varkappa}$ and a tensor of the second rank $\psi^{\lambda\nu}$; namely

$$\left(-\Delta+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\mu^2\right)\begin{pmatrix}\psi^{\varkappa}\\ \psi^{\lambda\nu}\end{pmatrix}=0 \qquad (1)$$

I showed in the previous paper[1] that these second order equations might be obtained by combining a system of the first order equations such as

$$\left.\begin{aligned} J^{\sigma\varkappa}_{\cdot\cdot\lambda\nu}\nabla_{\sigma}\psi^{\lambda\nu}+i\mu\psi^{\varkappa}=0\\ J^{\sigma\cdot\lambda\nu}_{\cdot\varkappa}\nabla_{\sigma}\psi^{\varkappa}+i\mu\psi^{\lambda\nu}=0\end{aligned}\right\} \qquad (2)$$

1. Jap. Jour. Phys. XI (1936), 35.
2. Jap. Jour. Phys. XII (1938), 27.
3. Proc. Phys. Math. Soc. Japan, 20 (1938), 321.

where $J_{\sigma\kappa\lambda\nu}$ is generally defined by

$$J_{\sigma\kappa\lambda\nu}=\frac{1}{2}(g_{\sigma\kappa}g_{\lambda\nu}+g_{\sigma\lambda}g_{\kappa\nu}-g_{\sigma\nu}g_{\kappa\lambda}+E_{\sigma\kappa\lambda\nu}) \qquad (3)$$

$g_{\sigma\kappa}$: metrical tensor

$E_{\sigma\kappa\lambda\nu}$: unit antisymmetrical tensor of the fourth rank,

or : $=i\sqrt{-g}\,\varepsilon_{\sigma\kappa\lambda\nu}$ (g stands for the determinant of $g_{\sigma\kappa}$ and $\varepsilon_{\sigma\kappa\lambda\nu}$ is the coefficient of the determinent.)

∇_σ : Riemannian covariant differentiation.

Since the equations of (1) are special-relativistic ones, we have to consider the case when the values of $g_{\sigma\kappa}$ are given by

$$g_{\sigma\kappa}=\begin{pmatrix}-1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1\end{pmatrix},\quad -g=1 \text{ and } \nabla_\sigma=\partial_\sigma\left(\equiv\frac{\partial}{\partial x^\sigma}\right).$$

I showed in the previous paper[1] that in the case of special relativity the system of equations (2) is equivalent to the equation of the Dirac electron.

As $E_{\sigma\kappa\lambda\nu}$ is a purely imaginary quantity, the tensor $J_{\sigma\kappa\lambda\nu}$ may be considered as a kind of complex quantity, and we may introduce its conjugate by

$$\bar{J}_{\sigma\kappa\lambda\nu}=\frac{1}{2}(g_{\sigma\kappa}g_{\lambda\nu}+g_{\sigma\lambda}g_{\kappa\nu}-g_{\sigma\nu}g_{\kappa\lambda}-E_{\sigma\kappa\lambda\nu}). \qquad (3')$$

Using this conjugate tensor, we may consider the following system of equations :

$$\left.\begin{aligned}\bar{J}^{\sigma\kappa}_{\cdot\cdot\lambda\nu}\nabla_\sigma\chi^{\lambda\nu}+i\mu\chi^\kappa=0,\\ \bar{J}^{\sigma\cdot\lambda\nu}_{\cdot\kappa}\nabla_\sigma\chi^\kappa+i\mu\chi^{\lambda\nu}=0,\end{aligned}\right\} \qquad (2')$$

which can also be reduced to the second order equations of the form (1). Thus, we have in general two systems of the first order equations which may correspond to the second order wave equations of the form (1).

On the other hand, in separating a real number μ^2, we may introduce a pair of complex conjugate numbers $\varkappa$ and $\bar{\varkappa}$, so that

$$\mu^2=\varkappa\bar{\varkappa}.$$

If we use this separation, it may also be possible to separate the first order equations into the following systems :

$$\left.\begin{aligned}J^{\sigma\kappa}_{\cdot\cdot\lambda\nu}\nabla_\sigma\psi^{\lambda\nu}+i\varkappa\psi^\kappa=0\\ J^{\sigma\cdot\lambda\nu}_{\cdot\kappa}\nabla_\sigma\psi^\kappa+i\bar{\varkappa}\psi^{\lambda\nu}=0\end{aligned}\right\} \qquad (4)$$

and

$$\left.\begin{aligned}\bar{J}^{\sigma\kappa}_{\cdot\cdot\lambda\nu}\nabla_\sigma\chi^{\lambda\nu}+i\varkappa\chi^\kappa=0\\ \bar{J}^{\sigma\cdot\lambda\nu}_{\cdot\kappa}\nabla_\sigma\chi^\kappa+i\bar{\varkappa}\chi^{\lambda\nu}=0\end{aligned}\right\} \qquad (4')$$

Now, let $\varkappa_1$ and $\varkappa_2$ be respectively the real and the imaginary part of $\varkappa$, so that

$$\varkappa=\varkappa_1+i\varkappa_2.$$

If we put $\varkappa_2=0$ and $\varkappa_1=\mu$ the equations (4) becomes (2), and the equations (4′) can be identified with the conjugate equations of (4) by the relations

$$\bar{\psi}^{\varkappa}=i\chi^{\varkappa}, \quad \bar{\psi}^{\lambda\nu}=-i\chi^{\lambda\nu}.$$

We may, therefore, identify $\varkappa_1$ with $\frac{mc}{\hbar}$ of the Dirac electron.

We shall next show the possibility of identifying $\varkappa_2$ with $\frac{m_u C}{\hbar}$ of the so-called meson.

Let us suppose $\varkappa_1=0$. We have then the possibility of putting $\psi^{\varkappa}=\chi^{\varkappa}$ as follows:

We write $\psi^{\varkappa}=\chi^{\varkappa}\equiv U^{\varkappa}$,

$$\psi^{\lambda\nu}+\chi^{\lambda\nu}\equiv F^{\lambda\nu},$$

$$\psi^{\lambda\nu}-\chi^{\lambda\nu}\equiv \tilde{F}^{\lambda\nu}.$$

Then we can reduce the equations (4) and (4′) to the following forms:

$$\begin{aligned} &g_{\lambda\nu}\nabla_\sigma U^\sigma+2\nabla_{[\lambda}U_{\nu]}=\varkappa_2 F_{\lambda\nu},\\ &E^{\sigma\tau}_{\lambda\nu}\nabla_\sigma U_\tau=\varkappa_2\tilde{F}_{\lambda\nu},\\ &\nabla_\lambda F^{\lambda\nu}+\varkappa_2 U^\nu=0. \end{aligned} \qquad (5)$$

Furthermore we assume $\nabla_\sigma U^\sigma=0$, namely, the divergence of vector U^σ vanishes. Then the first equation of (5) means that $F_{\lambda\nu}$ is a rotation (four-dimensional) of the vector U_ν, and the second equation of (5) means that $\tilde{F}_{\lambda\nu}$ is the dual tensor of $F_{\lambda\nu}$.

Let us now use the usual vector notation:

$$(F^{14}, F^{24}, F^{34})=\vec{F}, \quad (F_{23}, F_{31}, F_{12})=\vec{G},$$

$$(U^1, U^2, U^3)=\vec{U}, \quad U_0=U_4.$$

Then, the equations (5) can be written in the forms:

$$\frac{1}{C}\frac{\partial\vec{F}}{\partial t}-\mathrm{rot}\,\vec{G}-\varkappa_2\vec{U}=0, \quad \mathrm{div}\,\vec{F}+\varkappa_2 U_0=0,$$

$$\frac{1}{C}\frac{\partial\vec{U}}{\partial t}+\mathrm{grad}\,U_0+\varkappa_2\vec{F}=0, \quad \mathrm{rot}\,\vec{U}-\varkappa_2\vec{G}=0,$$

$$\frac{1}{C}\frac{\partial\vec{G}}{\partial t}+\mathrm{rot}\,\vec{F}=0, \quad \mathrm{div}\,\vec{G}=0, \quad \frac{1}{C}\frac{\partial U_0}{\partial t}+\mathrm{div}\,\vec{U}=0.$$

These are the equations of the so-called U-field proposed by Yukawa[3]. We may thus arrive at the possibility of identifying $\varkappa_2$ with $\frac{m_u C}{\hbar}$ of the meson.

If we introduce the electro-magnetic field in the usual way, namely

by substituting $\left(\partial_\mu + i\frac{e}{\hbar}\varphi_\mu\right)$ for ∂_μ in the equations of (4) and (4′), we may establish some connection between the fields ψ and χ.

In conclusion I wish to express my cordial thanks to Professor Niels Bohr through whose kindness I was given the opportunity of discussing this problem with prominent physicists of the University of Copenhagen.

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CONTENTS

A Method of Measuring Small Capacities by A Neon Tube
By Tatsumi Terada 33
A Note on Equations for the Meson
By Hideo Yamamoto 38